superintendence, of the transformed equation for the quintic equation. The several results are given in the present memoir; and for greater completeness, I reproduce the demonstration which I have given in the former of the above-mentioned two notes, of the general property, that the function of y is an invariant. At the end of the memoir I consider the problem of the reduction of the general quintic equation to Mr. Jerrard's form $x^5+ax+b=0$.

December 12, 1861.

Major-General SABINE, R.A., President, in the Chair.

In accordance with the announcement made from the Chair at the last Meeting, the question of Mr. Sievier's readmission was put to the vote, and was decided in the affirmative. The President accordingly declared that Mr. Sievier was readmitted into the Society.

The following communications were read:—

I. "On a Series for calculating the Ratio of the Circumference of a Circle to its Diameter." By AMOS CLARKSON, Esq. Communicated by Professor STOKES, Sec. R.S. Received September 27, 1861.

The ratio (π) of the circumference to the diameter of a circle may be calculated by the following series:—

$$\pi=\frac{8}{3}\left\{1-\frac{1}{3.10}-\frac{2}{3.5.10^2}-\frac{2.4}{3.5.7.10^3}-\dots\right\}$$
$$+\frac{4}{7}\left\{1-\frac{2}{3.10^2}-\frac{2.2^2}{3.5.10^4}-\frac{2.4.2^3}{3.5.7.10^6}-\dots\right\}. \quad . \quad (1)$$

This series may be thus established. We have, as is well known,

$$\frac{\pi}{4}=2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7};$$

and denoting by c the arc of which the tangent is t,

$$c=t-\frac{t^3}{3}+\frac{t^5}{5}-\frac{t^7}{7}+\dots$$

Put

$$t^2=\frac{1}{x-1},$$

then

$$c=\frac{1}{(x-1)^{\frac{1}{2}}}\left\{1-\frac{1}{3(x-1)}+\frac{1}{5(x-1)^2}-\frac{1}{7(x-1)^3}+\cdots\right\}$$

and

$$-\frac{1}{3(x-1)}=-\frac{1}{3}\left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}+\ldots\right),$$

$$\frac{1}{5(x-1)^2}=\frac{1}{5}\left(\frac{1}{x^2}+\frac{2}{x^3}+\frac{3}{x^4}+\ldots\right),$$

$$-\frac{1}{7(x-1)^3}=-\frac{1}{7}\left(\frac{1}{x^3}+\frac{3}{x^4}+\ldots\right),$$

$$\frac{1}{9(x-1)^4}=\frac{1}{9}\left(\frac{1}{x^4}+\ldots\right);$$

whence we get by addition,

$$c=\frac{1}{(x-1)}\left\{1-\frac{1}{3x}-\frac{2}{3\,.\,5x^2}-\frac{2\,.\,4}{3\,.\,5\,.\,7x^3}-\ldots\right\}. \quad . \quad (2)$$

The law of the coefficients may be discovered by induction, but is not easily demonstrated in this manner. It may be obtained as follows:—

We have by differentiation

$$dc=\frac{dt}{1+t^2}=-\frac{dx}{2x(x-1)^{\frac{1}{2}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Assume

$$-\frac{dx}{2x(x-1)^{\frac{1}{2}}}=d\left\{\frac{1}{(x-1)^{\frac{1}{2}}}\left[A+\frac{B}{x}+\frac{C}{x^2}+\frac{D}{x^3}+\cdots\right]\right\}; \quad (4)$$

then

$$-\frac{dx}{(x-1)^{\frac{3}{2}}}\left(\frac{x-1}{x}\right)=-\frac{dx}{(x-1)^{\frac{3}{2}}}\left\{\begin{array}{l}A+\frac{B}{x}+\frac{C}{x^2}+\frac{D}{x^3}+\cdots \\ +2(x-1)\left(\frac{B}{x^2}+\frac{2C}{x^3}+\frac{3D}{x^4}+\cdots\right.\end{array}\right\}$$

or

$$1-\frac{1}{x}=A+\frac{3B}{x}+\frac{5C}{x^2}+\frac{7D}{x^3}+\cdots$$
$$-\frac{2B}{x^2}-\frac{4C}{x^3}-\cdots$$

and by equating coefficients,

$$
\begin{aligned}
A &= 1, \\
B &= -\frac{1}{3}, \\
C &= \frac{2}{5}B = -\frac{2}{3 \cdot 5}, \\
D &= \frac{4}{7}C = -\frac{2 \cdot 4}{3 \cdot 5 \cdot 7}, \text{ \&c.};
\end{aligned}
$$

whence substituting in (4) and (3), integrating, and observing that $c=0$ when $t=0$ or $x=\infty$, we have the series (2). On substituting now in succession $\tan^{-1}\frac{1}{3}$ and $\tan^{-1}\frac{1}{7}$ for c, and therefore 10 and 50 for x, and in the latter case multiplying the numerators and denominators of the successive terms by successive powers of 2, we obtain the series (1).

These series, which the author believes to be new, follow a simple law, and converge with great rapidity. But their distinctive feature, compared with other series which have been given for the same object, consists in the fact that the denominators involve the successive powers of 10, the division by which is effected at once.

II. "On the Production of Vibrations and Sounds by Electrolysis." By George Gore, Esq. Communicated by Professor Tyndall. Received November 12, 1861.

(Abstract.)

In this communication, which is a continuation in subject (but different in title) of a previous investigation "On the Movements of Liquid Metals and Electrolytes in the Voltaic Circuit," the author has described the most convenient and effective method of obtaining vibrations and sounds by electrolysis.

The paper contains a full account of the influence of various circumstances upon the vibrations and sounds: viz., of the electrodes,—the electrolyte,—mechanical circumstances and temperature,—the electric current,—size and number of voltaic elements,—quantity of the current,—coils of wire in the circuit,—induction coils and iron cores,—electrolytes in the circuit,—and of magnetism: also the influence of the vibrating medium itself upon the electric current.

The best liquid for producing the vibrations and sounds consists of